

South Hylton Primary School



School Calculation Policy

Contents

Page Number

3	Introduction, Calculation Aims and Reasons for Using Written Methods
4	Whole School Approach
5	When Are Children Ready for Written Calculations?
6	Stages in Addition
15	Stages in Subtraction
25	Stages in Multiplication
32	Stages in Division

Introduction

The new national curriculum for mathematics aims to ensure that all pupils:

- become **fluent** in the fundamentals of mathematics, including through varied and frequent practice with increasingly complex problems over time, so that pupils develop conceptual understanding and the ability to recall and apply knowledge rapidly and accurately
- **reason mathematically** by following a line of enquiry, conjecturing relationships and generalisations, and developing an argument, justification or proof using mathematical language
- can **solve problems** by applying their mathematics to a variety of routine and non-routine problems with increasing sophistication, including breaking down problems into a series of simpler steps and persevering in seeking solutions.

The new curriculum has a strong emphasis on mental and written calculation of whole numbers, decimals and fractions.

Calculation aims

- Children are taught and acquire secure mental methods of calculation.
- Children are taught one efficient written method for addition, one efficient written method for subtraction, one efficient written method for multiplication and one efficient written method for division which they know and can rely on when mental methods are not appropriate.
- Mental and written methods of calculation should be applied frequently to relevant problem-solving contexts.

Reasons for using written methods

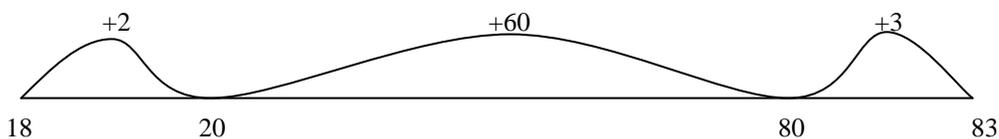
- To aid mental calculation by writing down some of the numbers and answers involved
- To make clear mental procedure
- To help communicate methods and solutions
- To provide a record of work to be done
- To aid calculation when the problem is too difficult to be done mentally
- To develop and refine a set of rules for calculation

Whole school approach

We have developed a consistent approach to the teaching of written calculation methods. This will establish continuity and progression throughout the school.

Mental methods will be established. These will be based on a solid understanding of place value in number and will include the following:

- i Remembering number facts and recalling them without hesitation
e.g. pairs of numbers which make 10
Doubles and halves to 20
- ii Using unknown facts to calculate unknown facts
e.g. $6 + 6 = 12$ therefore $6 + 7 = 13$
 $24 + 10 = 34$ therefore $24 + 9 = 33$
- iii Understanding and using relationships between addition and subtraction to find answers and check results
e.g. $14 + 6 = 20$ therefore $20 - 6 = 14$
- iv Having a repertoire of mental strategies to solve calculations
e.g. doubles/near doubles
bridging 10/bridging 20
adding 9 by +10 and -1
- v Making use of informal jottings such as blank number lines to assist in calculations with larger numbers *e.g. $83 - 18 = 65$*



- vi Solving one-step word problems (either mentally or with jottings) by identifying which operation to use, drawing upon their knowledge of number bonds and explaining their reasoning
- vii Beginning to present calculations in a horizontal format and explain mental steps using numbers, symbols or words
- viii Learn to estimate/approximate first e.g. $29 + 30$ (round up to nearest 10, the answer will be near to 60).

Place value will be taught mentally first from Reception class where number tracks are used, progressing to number lines (to 10 or 20 as appropriate) in Years 1 and 2. The empty number line will then be introduced to aid calculations.

Subtraction will be taught by counting on and counting back depending on the numbers.

Numbers such as 10, 100, 1000 will be called Landmark Numbers

When are children ready for written calculations?

Addition and subtraction

- Do they know addition and subtraction facts to 20?
- Do they understand place value and can they partition numbers?
- Can they add three single digit numbers mentally?
- Can they add and subtract any pair of two-digit numbers mentally?
- Can they explain their mental strategies orally and record them using informal jottings?

Multiplication and division

- Do they know the 2, 3, 4, 5 and 10 times tables?
- Do they know the result of multiplying by 0 and 1?
- Do they understand 0 as a placeholder?
- Can they multiply two and three-digit numbers by 10 and 100?
- Can they double and halve two-digit numbers mentally?
- Can they use multiplication facts they know to derive mentally other multiplication facts that they do not know?
- Can they explain their mental strategies orally and record them using informal jottings?

The above lists are not exhaustive but are a guide for the teacher to judge when a child is ready to move from informal to formal methods of calculation.

Stages in Addition

Stage 1: Count all

Curriculum Objectives: Foundation Stage (Early Learning Goals)

- Children count reliably with numbers from one to 20, place them in order and say which number is one more or one less than a given number.
- Using quantities and objects, they add and subtract two single-digit numbers and count on or back to find the answer.
- They solve problems, including doubling, halving and sharing.

Mental method:

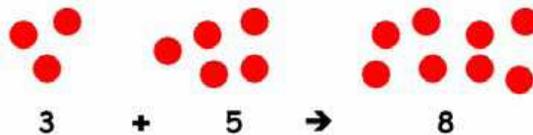
Count out the groups, then find the total by counting all the counters

Opportunities for practice:

Use practical resources in playful and relevant context.

Example of Written Method:

$$3 + 5 = 8$$



Stage 2: Count on from the first number

Curriculum Objectives: Foundation Stage (Early Learning Goals)

- Children count reliably with numbers from one to 20, place them in order and say which number is one more or one less than a given number.
- Using quantities and objects, they add and subtract two single-digit numbers and count on or back to find the answer.
- They solve problems, including doubling, halving and sharing.

Mental method:

Verbally 'say' the first number, and then use fingers to count on.

Demonstrate on a number line.

Opportunities for practise:

Practise addition strategy in playful situations and relevant contexts

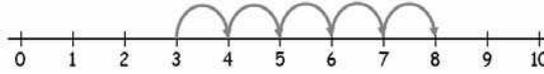
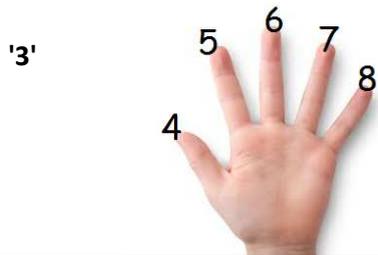
Number bonds to 10

Doubles to 5 (Double 1, 2, 3, 4, 5)

Estimating the answer

Example of Written Method:

$3+5=8$



Stage 3: Count on from the larger number

Curriculum Objectives: Year 1 (Addition and Subtraction)

- represent and use number bonds and related subtraction facts within 20
- add and subtract 1-digit and 2-digit numbers to 20, including zero
- read, write and interpret mathematical statements involving addition (+), subtraction (-) and equals (=) signs
- solve one-step problems that involve addition and subtraction, using concrete objects and pictorial representations, and missing number problems such as $7 = \square - 9$

Mental method:

Children decide which number is the largest, verbally 'say' this number, and use fingers to count on. Demonstrate on a number line.

Written method:

Write the number sentence.

Opportunities for practice:

Use practical resources in playful and relevant context, e.g.

- Snack time
- Tidy-up time
- Registration
- Play activities

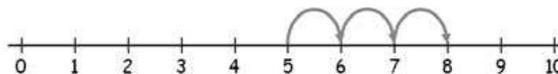
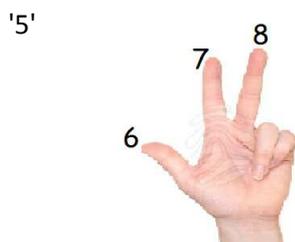
Number bonds up to 20

Doubles up to 10

Estimating the answer

Example of Written Method:

$5+3=8$



Include addition of 1 and 2 digit numbers to 20:

$9+9= 18$

Add three 1-digit numbers:

$$5+3+4=12$$

Stage 4: Addition on a hundred square

Curriculum Objectives: Year 1 (Addition and Subtraction)

- represent and use number bonds and related subtraction facts within 20
- add and subtract 1-digit and 2-digit numbers to 20, including zero
- read, write and interpret mathematical statements involving addition (+), subtraction (-) and equals (=) signs
- solve one-step problems that involve addition and subtraction, using concrete objects and pictorial representations, and missing number problems such as $7 = \square - 9$

Mental method:

Introduce a hundred square when the second number is larger than 10.

Count on 2 tens then 3 ones.

Written method:

Write the number sentence

Opportunities for practice:

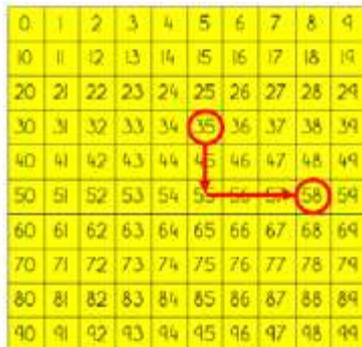
Number bonds up to 20

Estimating the answer

Relevant problem solving opportunities

Example of Written Method:

$$35+23=58$$



Stage 5: Counting on using a number line

Curriculum Objectives: Year 2 (Addition and Subtraction)

- recall and use addition and subtraction facts to 20 fluently, and derive and use related facts up to 100
- add and subtract numbers using concrete objects, pictorial representations, and mentally, including:
 - a two-digit number and units
 - a two-digit number and tens
 - two two-digit numbers
 - adding three one-digit numbers
- show that addition of two numbers can be done in any order (commutative) and subtraction of one number from another cannot
- add and subtract numbers with up to two digits, using formal written methods of columnar addition and subtraction
- recognise and use the inverse relationship between addition and subtraction and use this to check calculations and missing number problems.
- solve problems with addition and subtraction:
 - * using concrete objects and pictorial representations, including those involving numbers, quantities and measures
 - * applying their increasing knowledge of mental and written methods
- solve simple problems in a practical context involving addition and subtraction of money of the same unit, including giving change

Mental method:

Partitioning- adding the tens and ones separately

Written method:

Steps in addition can be recorded on a number line.

The steps often bridge through a multiple of 10.

Number line helps record the steps on the way to calculating the total.

Extend to column addition without

Opportunities for practice:

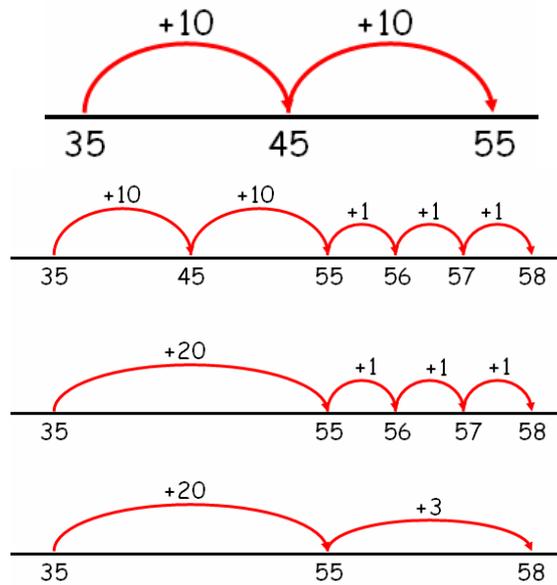
Use Dienes apparatus

Estimating the answer

Relevant problem solving opportunities

Example of Written Method:

$$35 + 20 = 55$$



Column addition (without carrying):

$$\begin{array}{r} 12 + \\ 5 \\ \hline 17 \end{array}$$

Stage 6: Partitioning leading to column addition

Curriculum Objectives: Year 3 (Addition and Subtraction)

- add and subtract numbers mentally, including:
 - a three-digit number and units
 - a three-digit number and tens
 - a three-digit number and hundreds
- add and subtract numbers with up to three digits, using formal written methods of columnar addition and subtraction
- estimate the answer to a calculation and use inverse operations to check answers
- solve problems, including missing number problems, using number facts, place value, and more complex addition and subtraction.

Mental method:

Step 1: Add the tens then the ones to form partial sums

Step 2: Add the partial sums

Written method:

Record steps in addition using partitioning.

Progress to column addition.

Opportunities for practice:

Estimating the answer

Relevant problem solving opportunities

Example of Written Method:

$18 + 15 = 33$	$35 + 23 = 58$
$10 + 10 = 20$	$30 + 20 = 50$
$8 + 5 = 13$	$5 + 3 = 8$
$20 + 13 = 33$	$50 + 8 = 58$

Column addition up to 3 digits:

$24 +$	$126 +$	237
<u>15</u>	<u>33</u>	<u>56</u>
<u>39</u>	<u>159</u>	<u>293</u>
		1

Stage 7: Column addition

Curriculum Objectives: Year 4 (Addition and Subtraction)

- add and subtract numbers mentally, including:
 - * a four-digit number and ones
 - * a four-digit number and tens
 - * a four-digit number and hundreds
 - * a four-digit number and thousands
- begin to add and subtract decimals to one decimal place
- add and subtract numbers with up to 4 digits using the efficient written methods of columnar addition and subtraction where appropriate
- estimate and use inverse operations to check answers to a calculation
- solve addition and subtraction two-step problems in contexts, deciding which operations and methods to use and why.

Mental method:

Step 1: Put the largest number first

Step 2: Add the tens

Step 3: Add the units

Written method:

Extend column addition up to 4 digits.

Extend to decimals with one decimal place

Opportunities for practice:

Estimating the answer

Relevant problem solving opportunities

Example of Written Method:

Column addition with up to 4 digits.

$$\begin{array}{r} 2435 + \\ 3659 \\ \hline 6094 \\ \hline \begin{array}{cc} 1 & 1 \end{array} \end{array}$$

Column addition with decimals:

$$\begin{array}{r} 56.3 + \\ 2.6 \\ \hline 58.9 \\ \hline \end{array}$$

Stage 8: Column addition (continued)

Curriculum Objectives: Year 5 (Addition and Subtraction)

- add and subtract numbers mentally with increasingly large numbers
- add and subtract whole numbers with more than 4 digits, including using formal written methods (columnar addition and subtraction)
- begin to add and subtract decimals with more than one decimal place, which involve carrying.
- use rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy
- solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why.

Mental method:

Children place numbers in columns, drawing on their understanding of place value.

Written method:

Column addition remains efficient when used with larger whole numbers and decimals. Once learned, the method is quick and reliable.

Carry digits are recorded under the line, using the words: 'carry ten' or 'carry one hundred', not 'carry one'.

Add numbers with more than 4 digits.

Continue to add numbers to one decimal place, including carrying

Extend to adding numbers with more than one decimal place, including carrying.

Opportunities for practice:

Estimating the answer

Relevant problem solving opportunities.

Example of Written Method:	
Column addition with 5 digits:	Column addition with decimals:
$\begin{array}{r} 15362 + \\ 23456 \\ \hline 38818 \end{array}$	$\begin{array}{r} 56.3 + \\ \underline{2.6} \\ 58.9 \end{array}$
1	
Column addition with decimals including carrying	
$\begin{array}{r} 56.5 + \\ 2.6 \\ \hline 59.1 \end{array}$	
1	

Stage 9: Continue to practise and apply skills

Curriculum Objectives: Year 6 (Addition and Subtraction)
<ul style="list-style-type: none"> • Perform mental calculations, including with mixed operations and large numbers • Use their knowledge of the order of operations to carry out calculations involving the four operations • add and subtract whole numbers with more than 4 digits, including using formal written methods (columnar addition and subtraction) • Use estimation to check answer to calculations and determine, in the context of a problem, levels of accuracy. • Solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why • Solve problems involving addition, subtraction, multiplication and division
<p>Mental method: Children should be able to perform mental calculations, including with mixed operations and large numbers.</p> <p>Written method: Column addition involving larger numbers.</p> <p>Opportunities for practice: Children should apply the formal written method to word and real-life problems.</p>

Example of Written Method:

Use column addition in problem solving contexts.

Continue to practise column addition and extend to large numbers when appropriate:

$$\begin{array}{r} 15362 + \\ 234156 \\ \hline 38818 \end{array}$$

Column addition with decimals:

$$\begin{array}{r} 56.3 + \\ 2.6 \\ \hline 58.9 \end{array}$$

Column addition with decimals including carrying

$$\begin{array}{r} 56.5 + \\ 2.6 \\ \hline 59.1 \end{array}$$

1

Stages in Subtraction

Stage 1: Counting back (take away reduction)

<p>Curriculum Objectives: Foundation Stage (Early Learning Goals)</p> <ul style="list-style-type: none"> • Children count reliably with numbers from one to 20, place them in order and say which number is one more or one less than a given number. • Using quantities and objects, they add and subtract two single-digit numbers and count on or back to find the answer. • They solve problems, including doubling, halving and sharing.
<p>Mental method: Count how objects need to be ‘taken away’, physically moving them or crossing out pictures. Count how many are left.</p> <p>Opportunities for practice: Use practical resources in playful and relevant context</p>
<p>Example of Written Method:</p> <p>$13 - 5 = 8$</p>   <p>$10 - 4 = 6$</p>

Stage 2: Counting back using fingers

<p>Curriculum Objectives: Foundation Stage (Early Learning Goals)</p> <ul style="list-style-type: none"> • Children count reliably with numbers from one to 20, place them in order and say which number is one more or one less than a given number. • Using quantities and objects, they add and subtract two single-digit numbers and count on or back to find the answer. • They solve problems, including doubling, halving and sharing.
<p>Mental method: Verbally ‘say’ the largest number, and then use fingers to count back. Demonstrate on a number line.</p> <p>Opportunities for practice: Practise subtraction strategy in playful situations and relevant contexts. Number bonds to 10 Doubles to 5 (double 1, 2, 3, 4, 5) Estimating the answer</p>

Example of Written Method:

$$9 - 3 = 6$$



$$10 - 4 = 6$$



Stage 3: Count on from the smaller number

Curriculum Objectives: Year 1 (Addition and Subtraction)

- represent and use number bonds and related subtraction facts within 20
- add and subtract 1-digit and 2-digit numbers to 20 ($9 + 9$, $18 - 9$), including zero
- Read, write and interpret mathematical statements involving addition (+), subtraction (-) and equals (=) signs.
- Solve one-step problems that involve addition and subtraction, using concrete objects and pictorial representations, and missing number problems such as $7 = \square - 9$.

Mental method:

Children decide which number is the largest, verbally 'say' this number, and use fingers to count on. Demonstrate on a number line.

Written method:

Write the number sentence.

Opportunities for practice:

Use practical resources in playful and relevant context.

Number bonds up to 20

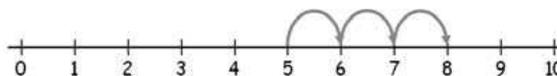
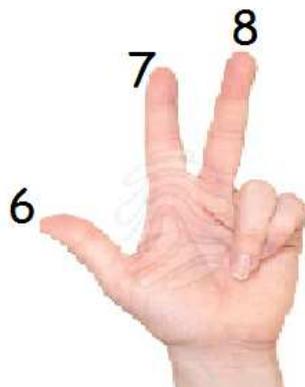
Doubles up to 10

Estimating the answer

Example of Written Method:

$$8 - 5 = 3$$

'5'



Stage 4: Counting on to find the difference

Curriculum Objectives: Year 1 (Addition and Subtraction)

- represent and use number bonds and related subtraction facts within 20
- add and subtract 1-digit and 2-digit numbers to 20 ($9 + 9$, $18 - 9$), including zero
- Read, write and interpret mathematical statements involving addition (+), subtraction (-) and equals (=) signs.
- Solve one-step problems that involve addition and subtraction, using concrete objects and pictorial representations, and missing number problems such as $7 = \square - 9$.

Mental method:

Step 1: Introduce using practical resources, verbally counting on 'How many more?'

Step 2: Find the difference using a number line. Verbally say the smallest number and count on to the largest number.

Step 3: Count up from the smallest number to the largest number.

e.g. Start at 27 and count on to 31

Written method:

Steps in subtraction can be recorded as 'jumps' on a number line.

The steps often bridge through a multiple of 10.

Opportunities for practice:

Use Dienes apparatus

Estimating the answer

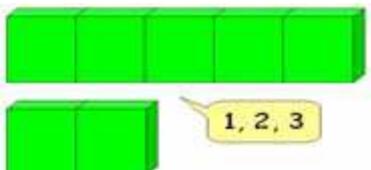
Number bonds and subtraction facts to 20

Partitioning two

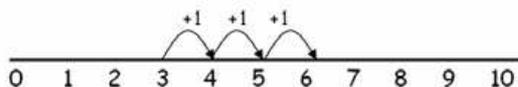
Relevant problem solving opportunities

Example of Written Method:

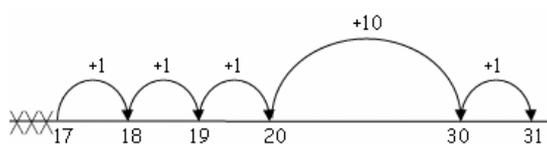
$$5 - 2 = 3$$



$$6 - 3 = 3$$



$$31 - 17 = 14$$



Stage 5: Introducing column subtraction

Curriculum Objectives: Year 2 (Addition and Subtraction)

- recall and use addition and subtraction facts to 20 fluently, and derive and use related facts up to 100
- add and subtract numbers using concrete objects, pictorial representations, and mentally, including:
 - a two-digit number and units
 - a two-digit number and tens
 - two two-digit numbers
 - adding three one-digit numbers
- show that addition of two numbers can be done in any order (commutative) and subtraction of one number from another cannot
- add and subtract numbers with up to two digits, using formal written methods of columnar addition and subtraction
- recognise and use the inverse relationship between addition and subtraction and use this to check calculations and missing number problems.
- Solve problems with addition and subtraction:
 - using concrete objects and pictorial representations, including those involving numbers, quantities and measures
 - applying their increasing knowledge of mental and written methods
- Solve simple problems in a practical context involving addition and subtraction of money of the same unit, including giving change

Mental method:

Reinforce place value by partitioning 2-digit numbers
 Subtraction on a hundred square, again reinforcing place value
 Introduce column subtraction, without exchanging.

Written method:

Record subtraction calculations initially in terms of partitioning, then introduce column subtraction.

Opportunities for practice:

- Use Dienes apparatus
- Estimating the answer
- Number bonds and subtraction facts to 20
- Partitioning 2-digit numbers
- Relevant problem solving opportunities

Example of Written Method:

Partitioning a 2-digit number:

$10 + 5 = 15$

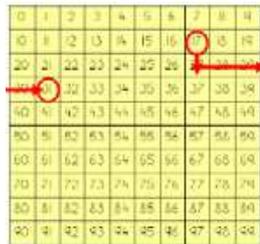
'One ten and five units make 15'

Subtraction on a hundred square:

$31 - 17 = 14$

4 units 'back' and one ten 'up'

$4 + 10 = 14$



Column subtraction (without exchanging):

TU – U

TU - TU

$18 -$

6

12

$18 -$

$\underline{12}$

$\underline{6}$

Stage 6: Recap partitioning and extend column subtraction

Curriculum Objectives: Year 3 (Addition and Subtraction)

- add and subtract numbers mentally, including:
 - a three-digit number and units
 - a three-digit number and tens
 - a three-digit number and hundreds
- add and subtract numbers with up to three digits, using the efficient written methods of columnar addition and subtraction
- estimate the answer to a calculation and use inverse operations to check answers
- solve problems, including missing number problems, using number facts, place value, and more complex addition and subtraction.

Mental method:

Subtract the tens then the units

Written method:

Record steps in subtraction using partitioning

Move to column subtraction with up to 3 digits, including exchanging.

Opportunities for practice:

Use Dienes apparatus

Estimating the answer

Number bonds and subtraction facts to 20

Partitioning 2 and 3-digit numbers

Relevant problem solving opportunities

Example of Written Method:

Subtraction by partitioning to reinforce place value:

$$58 - 23 \qquad 146 - 27$$

$$58 - 20 \qquad 146 - 20$$

$$38 - 3 \qquad 126 - 7$$

$$35 \qquad 119$$

Introduce column subtraction with up to 3- digits. Include some exchanging:

TU - U

1 1

$$\begin{array}{r} 26 \\ - 9 \\ \hline \end{array}$$

$$\begin{array}{r} 9 \\ \hline \end{array}$$

$$\begin{array}{r} 17 \\ \hline \end{array}$$

HTU - TU

3 1

$$\begin{array}{r} 146 \\ - 37 \\ \hline \end{array}$$

$$\begin{array}{r} 37 \\ \hline \end{array}$$

$$\begin{array}{r} 109 \\ \hline \end{array}$$

Stage 7: Counting on, find a missing number

Curriculum Objectives: Year 3 (Addition and Subtraction)

- add and subtract numbers mentally, including:
 - a three-digit number and units
 - a three-digit number and tens
 - a three-digit number and hundreds
- add and subtract numbers with up to three digits, using the efficient written methods of columnar addition and subtraction
- estimate the answer to a calculation and use inverse operations to check answers
- solve problems, including missing number problems, using number facts, place value, and more complex addition and subtraction.

Mental method:

Count up from the smallest number to the largest number.

Reduce the number of jumps by combining steps.

Written method:

Show jumps on an empty number line

Write the number sentence

Opportunities for practice:

Partitioning

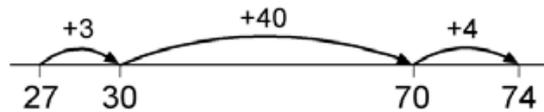
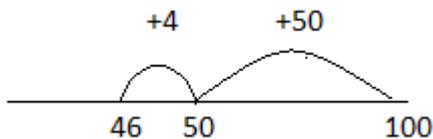
Addition and subtraction facts to 20

The counting-up method can be a useful alternative for children whose progress is slow, whose mental and written calculation skills are weak and whose projected attainment at the end of Key Stage 2 is towards the lower end of level 4.

Example of Written Method:

$100 - 46 = 54$

$74 - 27 = 47$



Stage 8: Column subtraction

Curriculum Objectives: Year 4 (Addition and Subtraction)

- add and subtract numbers mentally, including:
 - * a four-digit number and ones
 - * a four-digit number and tens
 - * a four-digit number and hundreds
 - * a four-digit number and thousands
- begin to add and subtract decimals to one decimal place
- add and subtract numbers with up to 4 digits using the efficient written methods of columnar addition and subtraction where appropriate
- estimate and use inverse operations to check answers to a calculation
- solve addition and subtraction two-step problems in contexts, deciding which operations and methods to use and why.

Mental method:

Children place numbers in columns, drawing on their understanding of place value.

Written method:

Column subtraction with up to 4 digits, including exchanging.

Extend to decimals with one decimal place

Opportunities for practice:

Use Dienes apparatus

Estimating the answer

Number bonds and subtraction facts to 20

Relevant problem solving opportunities

Example of Written Method:

Subtraction with up to 4-digits

ThHTU - HTU

$$\begin{array}{r}
 \overset{3}{} \overset{1}{} \\
 57\cancel{4}6 - \\
 \underline{637} \\
 5109
 \end{array}$$

Subtraction with up to one decimal place

$$\begin{array}{r}
 5.3 - \\
 \underline{2.2} \\
 3.2
 \end{array}$$

Stage 9: Column subtraction (continued)

Curriculum Objectives: Year 5 (Addition and Subtraction)

- add and subtract numbers mentally with increasingly large numbers
- add and subtract whole numbers with more than 4 digits, including using formal written methods (columnar addition and subtraction)
- Continue to add and subtract decimals with to one decimal place, which involve carrying or exchanging.
- begin to add and subtract decimals with more than one decimal place, which involve exchanging.
- use rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy
- solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why.

Mental method:

Children place numbers in columns, drawing on their understanding of place value.

Written method:

Compact subtraction remains efficient when used with larger whole numbers and decimals. Once learned, the method is quick and reliable.

Opportunities for practice:

Estimating the answer

Relevant problem solving opportunities

Partitioning two, three and 4-digit numbers

Extend to partitioning numbers with more than 4 digits and decimals

Example of Written Method:

Subtraction with up to 5 digits

HThThHTU – ThHTU

$$\begin{array}{r}
 6 \quad 1 \quad 4 \quad 1 \\
 \cancel{7}26\cancel{5}4 - \\
 \underline{5627} \\
 67027
 \end{array}$$

Subtraction with up to one decimal place

$$\begin{array}{r}
 5.3 - \\
 \underline{2.2} \\
 3.2
 \end{array}$$

Subtraction with decimals including exchanging

$$\begin{array}{r}
 6 \quad 1 \\
 \cancel{17}.6 - \\
 \underline{12.8} \\
 \underline{4.8}
 \end{array}$$

Stage 10: Continue to practise and apply skills

Curriculum Objectives: Year 6 (Addition and Subtraction)

- Perform mental calculations, including with mixed operations and large numbers
- Use their knowledge of the order of operations to carry out calculations involving the four operations
- add and subtract whole numbers with more than 4 digits, including using formal written methods (columnar addition and subtraction)
- Use estimation to check answer to calculations and determine, in the context of a problem, levels of accuracy.
- Solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why
- Solve problems involving addition, subtraction, multiplication and division

Mental method:

Children should be able to perform mental calculations, including with mixed operations and large numbers.

Written method:

Column subtraction involving larger numbers.

Opportunities for practice:

Children should apply the formal written method to word and real-life problems.

Example of Written Method:

Subtraction with up to 5 digits

HThThHTU – ThHTU

$$\begin{array}{r}
 6 \quad 1 \quad 4 \quad 1 \\
 \cancel{7}26\cancel{5}4 - \\
 \underline{5627} \\
 67027
 \end{array}$$

Subtraction with up to one decimal place

$$\begin{array}{r}
 5.3 - \\
 \underline{2.2} \\
 3.2
 \end{array}$$

Subtraction with decimals including exchanging

$$\begin{array}{r}
 6 \quad 1 \\
 \cancel{17}.6 - \\
 \underline{12.8} \\
 4.8
 \end{array}$$

Stages in Multiplication

Stage 1: Counting in equal steps of 2

Curriculum Objectives: Foundation Stage (Early Learning Goals)

- Children count reliably with numbers from one to 20, place them in order and say which number is one more or one less than a given number.
- Using quantities and objects, they add and subtract two single-digit numbers and count on or back to find the answer.
- They solve problems, including doubling, halving and sharing.

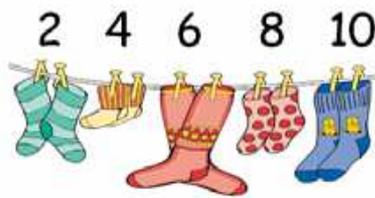
Mental method:

Children will begin to count in steps of 2s.

Opportunities for practice:

Use practical resources in playful and relevant context.

Example of Written Method:



Stage 2: Repeated addition

Curriculum Objectives: Year 1 (Multiplication and Division)

- Count in multiples of twos, threes, fours, fives and tens
- Solve simple one-step problems involving multiplication and division, calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.

Mental Method:

Children will experience equal groups of objects and will begin counting in 2s, 5s and 10s.

Written method:

Children write the groups of repeated addition, as shown in the diagram.

Recognise and write the 'x' sign in mathematical statements. Calculate the answer with the teacher using concrete objects.

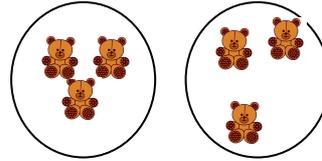
Opportunities for practice:

Practise multiplication strategy in playful situations and relevant contexts.

Example of Written Method:



$2 + 2 + 2 + 2 + 2 = 10$
 $2 \times 5 = 10$
 2 multiplied by 5
 5 pairs



$3 + 3 = 6$ $3 \times 2 = 6$ 3 multiplied by 2

Stage 3: Arrays

Curriculum Objectives: Year 2 (Multiplication and Division)

- Count in steps of 2, 3, 4, 5 and 10 from 0 and from any number forward or backward
- Recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers
- Show that multiplication of two numbers can be done in any order (commutative) and division of one number by another cannot
- Solve one-step problems involving multiplication and division, using materials, arrays, repeated addition, mental methods and multiplication and division facts, including problems in contexts.
- Calculate mathematical statements for multiplication and division within the multiplication tables and write them using the multiplication (x), division (÷) and equals (=) signs

Mental Method:

Following on from repeated addition, children use multiplication strategy of arrays. Count in equal steps, then relate to multiplication fact. Children may also count equal steps using their fingers.

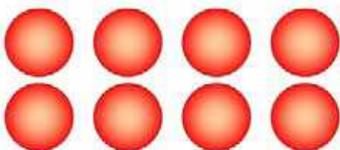
Written method:

Draw the array, then write multiplication sentence. Use the X and = signs to write mathematical statements. Ensure pupils recognise that multiplication can be done in any order.

Opportunities for practice:

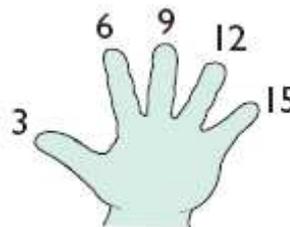
Estimating the answer
Times tables: 2, 5 and 10 up to x 12.

Example of Written Method:



$2 \times 4 = 8$

$4 \times 2 = 8$



Stage 4: Partitioning – Multiplication of TU x U

Curriculum Objectives: Year 3 (Multiplication and Division)

- Count in multiples of 4, 6, 8, 50 and 100
- Recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables
- Write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times on-digit numbers, using mental and progressing to formal written methods
- Estimate the answer to a calculation and use inverse operations to check answers
- Solve problems, including missing number problems, involving multiplication and division including integer scaling problems and correspondence problems in which n objects are connected to m objects

Mental method:

Step 1: TU number is partitioned and multiplied by the U.

Step 2: Totals are added together in column addition.

Written method:

Record steps in multiplication as shown in the diagram.

Opportunities for practice:

Estimating the answer

Relevant problem solving opportunities.

Partitioning two and three-digit numbers

Times tables: 2,3,4,5,8,10 up to x 12.

Example of Written Method:

$$37 \times 4 =$$

$$30 \times 4 = 120$$

$$7 \times 4 = 28$$

$$120 + 28 = 148$$

Stage 5: Short multiplication

Curriculum Objectives: Year 4 (Multiplication and Division)

- Count in multiples of 7, 9, 11, 12, 25 and 1000
- Recall multiplication and division facts for multiplication tables up to 12 x 12
- Use place value, known and derived facts to multiply and divide mentally, including: multiplying by 0 and 1; dividing by 1; multiplying together three numbers
- Recognise and use factor pairs and commutativity in mental calculations
- Multiply and divide whole numbers and those involving decimals by 10, 100 and 1000.
- Multiply two-digit and three-digit numbers by a one-digit number using formal written layout
- Estimate and use inverse operations to check answers to a calculation.
- Solve problems involving multiplying and adding, including using the distributive law and harder multiplication problems such as which n objects are connected to m objects

Mental method:

Children place numbers in columns, drawing on their understanding of place value.

Written method:

Standard written method

Multiplication of TU x U.

Extend to HTU x U.

Opportunities for practice:

Estimating the answer

Relevant problem solving opportunities.

Times tables: Derive and recall all multiplication facts up to 12 x12.

Example of Written Method:

38 x 7

$$\begin{array}{r} 38 \\ \times 7 \\ \hline 266 \\ 5 \end{array}$$

Extending to HTU x U.

Stage 6: Long multiplication

Curriculum Objectives: Year 5 (Multiplication and Division)

- Count forwards or backwards in steps of powers of 10 for any given number up to 1 000 000.
- Multiply and divide numbers mentally drawing upon known facts
- Multiply numbers up to 4 digits by a one- or two-digit number using an efficient written method, including long multiplication for two-digit numbers
- Identify multiples and factors, including finding all factor pairs
- Know and use the vocabulary of prime numbers, prime factors and composite (non-prime) numbers
- Establish whether a number up to 100 is prime and recall prime numbers up to 19
- Recognise and use square numbers and cube numbers, and the notation for squared (²) and cubed (³)
- Use estimation to check answers to calculations and determine, in the context of a problem, levels of accuracy.
- Solve problems involving multiplication and division including using their knowledge of factors and multiples, squares and cubes.
- Solve problems involving addition, subtraction, multiplication and division and a combination of these, including understanding the meaning of the equals sign.
- Solve problems involving multiplication and division, including scaling by simple fractions and problems involving simple rates.

Mental method:

Children place numbers in columns. Reinforce understanding of place value when multiplying by the ten.

Written method:

Record steps in multiplication as shown in the diagram.

Multiplication of TU x TU

Extend to HTU x TU

Opportunities for practice:

Estimating the answer

Relevant problem solving opportunities.

Times tables: Derive and recall all multiplication facts up to 12 x 12.

Example of Written Method:

$$56 \times 27$$

56×27 is approximately $60 \times 30 = 1800$.

$$\begin{array}{r}
 156 \\
 \times 27 \\
 \hline
 392 \\
 1120 \\
 \hline
 1512
 \end{array}$$

$$286 \times 29$$

286×29 is approximately $300 \times 30 = 9000$

$$\begin{array}{r} 12186 \\ X \quad 29 \\ \hline 2574 \\ 51720 \\ \hline 8294 \end{array}$$

Stage 7: Extension and consolidation of long multiplication – using and applying

Curriculum Objectives: Year 6 (Multiplication and Division)

- Perform mental calculations, including with mixed operations and large numbers
- Multiply multi-digit numbers up to 4 digits by a two-digit whole number using the efficient written method of long multiplication
- Identify common factors, common multiples and prime numbers
- Use their knowledge of the order of operations to carry out calculations involving the four operations
- Use estimation to check answers to calculations and determine, in the context of a problem, levels of accuracy
- Solve problems involving addition, subtraction, multiplication and division

Mental method:

Children place numbers in columns. Reinforce understanding of place value when multiplying by the ten.

Written method:

Record steps in multiplication as shown in the diagram.

Extend to ThHTU x TU

Opportunities for practice:

Estimating the answer

Relevant problem solving opportunities.

Times tables: Derive and recall all multiplication facts up to 12×12 .

Example of Written Method:

$$286 \times 29$$

286×29 is approximately $300 \times 30 = 9000$

$$\begin{array}{r} 1218\ 6 \\ X \quad \underline{7259} \\ \quad 2574 \\ \quad \underline{51720} \\ \quad 8294 \end{array}$$

Extend to ThHTU

Stages in Division

Stage 1: Sharing practically

Curriculum Objectives: Foundation Stage (Early Learning Goals)

- Children count reliably with numbers from one to 20, place them in order and say which number is one more or one less than a given number.
- Using quantities and objects, they add and subtract two single-digit numbers and count on or back to find the answer.
- They solve problems, including doubling, halving and sharing.

Mental method:

Children will begin to talk about sharing, using practical resources.

Opportunities for practice:

Use practical resources in playful and relevant context.

Example of Written Method:

Cut the pizza in half. How many pieces are there?



Stage 2: Sharing into groups

Curriculum Objectives: Year 1 (Multiplication and Division)

- Solve simple one-step problems involving multiplication and division, calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.

Mental method:

Children will begin to practically share objects into groups.

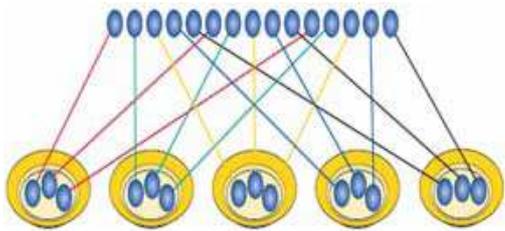
Use words: '5 groups of 3' or '2 groups of 3' etc.

Children will begin to recognise and write the \div symbol in mathematical statements, calculating the answer to word problems using practical objects and recording number sentence (with teacher support).

Opportunities for practice:

Use practical resources in playful and relevant context.

Example of Written Method:

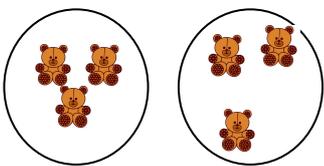


15 marbles are shared out equally among 5 children.

$$15 \div 5 = 3$$

6 teddies shared into 2 hoops.

$$6 \div 2 = 3$$



Stage 3: Using arrays

Curriculum Objectives: Year 2 (Multiplication and Division)

- Recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers
- Show that multiplication of two numbers can be done in any order (commutative) and division of one number by another cannot
- Calculate mathematical statements for multiplication and division within the multiplication tables and write them using the multiplication (x), division (÷) and equals (=) signs
- Solve one-step problems involving multiplication and division, using materials, arrays, repeated addition, mental methods and multiplication and division facts, including problems in contexts.

Mental method:

Following on from practical sharing, children should now be familiar with the ÷ sign and will write mathematical statements, calculating the answer to word problems using practical objects and recording number sentences.

Again use words: '5 groups of 3' or '2 groups of 3' etc.

Written method:

Draw the array, then write division sentence.

Use the ÷ and = signs to write mathematical statements.

Ensure pupils use inverse relations (e.g. $4 \times 5 = 20$ and $20 \div 5 = 4$).

Opportunities for practice:

Estimating the answer.

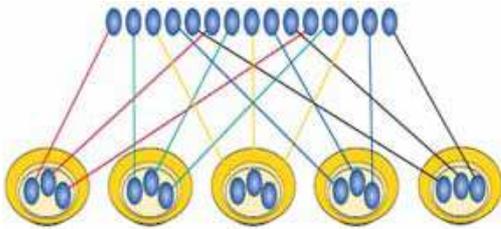
Relevant problem solving opportunities.

Times Tables: Recall of division facts for 2, 5 and 10 times tables

Example of Written Method:

15 marbles are shared out equally among 5 children.

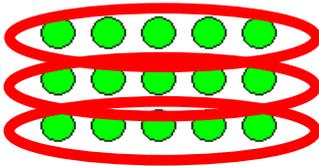
$$15 \div 5 = 3$$



Use of arrays: Sharing

The gardener planted 15 seeds in 3 equal rows.

How many seeds in each row? $15 \div 3 = 5$

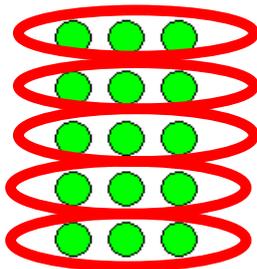


Grouping

The gardener planted 15 seeds with 3 seeds in each row.

How many rows of seeds are there?

$$15 \div 3 = 5$$



Stage 4: Grouping on a number line

Curriculum Objectives: Year 3 (Multiplication and Division)

- Recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables
- Write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times on-digit numbers, using mental and progressing to efficient written methods
- Estimate the answer to a calculation and use inverse operations to check answers.
- Solve problems, including missing number problems, involving multiplication and division including integer scaling problems and correspondence problems in which n objects are connected to m objects

Mental Method:

Recall multiplication and division facts for the 2, 3, 4, 5, 8 and 10 multiplication tables.

Written method:

Develop reliable written methods for division, starting with calculations of two-digit numbers by one-digit numbers and progressing to the efficient written method of short division.

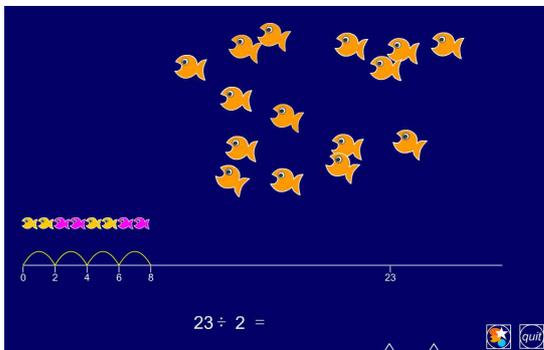
Opportunities for practice:

Relevant problem solving opportunities.

Times Tables: Recall of division facts for 2, 3, 4, 5, 8 and 10 times tables.

Example of Written Method:

$$14 \div 2 = 7$$



Stage 5: Short division

Curriculum Objectives: Year 4 (Multiplication and Division)

- Recall multiplication and division facts for multiplication tables up to 12 x 12
- Use place value, known and derived facts to multiply and divide mentally, including: multiplying by 0 and 1; dividing by 1; multiplying together three numbers
- Recognise and use factor pairs and commutativity in mental calculations
- Multiply and divide whole numbers and those involving decimals by 10, 100 and 1000.
- Multiply two-digit and three-digit numbers by a one-digit number using formal written layout
- Divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context.
- Estimate and use inverse operations to check answers to a calculation.
- Solve problems involving multiplying and adding, including using the distributive law and harder multiplication problems such as which n objects are connected to m objects

Mental Method:

Continue to practise recalling multiplication facts and related division facts to aid fluency.

Written method:

Short division $TU \div U$ as shown in the example.

Extend to $HTU \div U$.

Opportunities for practice:

Estimating the answer

Relevant problem solving opportunities.

Times Tables: Recall of division facts all tables up to 12 x 12.

Example of Written Method:

$$\begin{array}{r} 12 \\ 8 \overline{) 96} \end{array}$$

Extend to $HTU \div U$.

Stage 6: Extended short division

Curriculum Objectives: Year 5 (Multiplication and Division)

- Multiply and divide numbers mentally drawing upon known facts
- Multiply numbers up to 4 digits by a one- or two-digit number using an efficient written method, including long multiplication for two-digit numbers
- Identify multiples and factors, including finding all factor pairs
- Divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context. Extend to producing decimal answers.
- Know and use the vocabulary of prime numbers, prime factors and composite (non-prime) numbers
- Establish whether a number up to 100 is prime and recall prime numbers up to 19
- Recognise and use square numbers and cube numbers, and the notation for squared (²) and cubed (³)
- Use estimation to check answers to calculations and determine, in the context of a problem, levels of accuracy.
- Solve problems involving multiplication and division including using their knowledge of factors and multiples, squares and cubes.
- Solve problems involving addition, subtraction, multiplication and division and a combination of these, including understanding the meaning of the equals sign.
- Solve problems involving multiplication and division, including scaling by simple fractions and problems involving simple rates.

Mental Method:

Continue to practise recalling multiplication facts and related division facts to aid fluency.

Written method:

Continue to practise short division as shown in example.

TU ÷ U, HTU ÷ U, ThHTU ÷ U.

Opportunities for practice:

Estimating the answer

Interpreting remainders appropriately.

Relevant problem solving opportunities.

Times Tables: Recall of division facts all tables up to 12 x 12.

Example of Written Method:

Short division

$$\begin{array}{r} 12 \\ 8 \overline{) 96} \end{array}$$

Short division with remainders

$$\begin{array}{r} 13r1 \\ 6 \overline{) 79} \end{array}$$

Short division producing decimal answer

$$\begin{array}{r} 15.4 \\ 5 \overline{) 77.20} \end{array}$$

Stage 7: Long division

Curriculum Objectives: Year 5 (Multiplication and Division)

- Perform mental calculations, including with mixed operations and large numbers.
- Associate a fraction with division and calculate decimal fraction equivalents (e.g. 0.375) for a simple fraction (e.g. $\frac{3}{8}$).
- Divide numbers up to 4-digits by a two-digit whole number using the formal written method of short division where appropriate for the context. Produce decimal answers to x number of decimal places.
- Divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context.
- Identify common factors, common multiples and prime numbers.
- Use their knowledge of the order of operations to carry out calculations involving the four operations.
- Use estimation to check answers to calculations and determine, in the context of a problem, levels of accuracy.
- Solve problems involving addition, subtraction, multiplication and division.
- Solve problems involving similar shapes where the scale factor is known or can be found.

Mental Method:

Continue to practise recalling multiplication facts and related division facts to aid fluency.

Written method:

Continue short division as in year 5

Long division HTU \div TU.

Extend to ThHTU \div TU as shown in the example.

Opportunities for practice:

Estimating the answer

Relevant problem solving opportunities.

Times Tables: Recall of division facts all tables up to 12 x 12.

Example of Written Method:

Short division

$$\begin{array}{r} 12 \\ 8 \overline{) 96} \end{array}$$

Short division with remainders

$$\begin{array}{r} 13 \text{ r } 1 \\ 6 \overline{) 79} \end{array}$$

Short division producing decimal answer

$$\begin{array}{r} 15.4 \\ 5 \overline{) 77.20} \end{array}$$

Long Division

$$\begin{array}{r} 023 \\ 37 \overline{) 875} \\ \underline{-74} \\ 135 \\ \underline{-111} \\ 24 \end{array}$$

$$\begin{array}{r} 0573 \text{ r } 7 \\ 17 \overline{) 9748} \\ \underline{-85} \\ 124 \\ \underline{-119} \\ 58 \\ \underline{-51} \\ 7 \end{array}$$